NIELS BOHR INSTITUTE
UNIVERSITY OF COPENHAGEN

Spin squeezing on the caesium clock transition:

Interferometric quantum non-demolition measurement on cold atoms

Daniel Oblak

Danish National Research Foundation Centre for Quantum Optics (Quantop),
Niels Bohr Institute, University of Copenhagen

Experimentalists

Current

Ulrich Busck Hoff
Patrick Windpassinger
Daniel Oblak
Jürgen Appel
Niels Kjærgaard
Eugene Polzik

Previous

Mark Saffman
Carlos Garrido Alzar
Plamen Petrov
Jens Lykke Sørensen
Jens Mikkelsen
Anton Vershovski
Wolfgang Tittel

Overview of presentation

This presentation will contain:

- Motivation and applications
- Atomic system, interferometer and quantum variables
- QND interaction
- Experimental setup and characterisation
- Measurements of atomic noise
- Things to come (soon)

Motivation and applications

What's it all good for?

Increased sensitivity in spectroscopy[1,2,3]

- Atomic magnetometers
- Microwave fountain clocks
- Optical lattice clocks

Quantum memory[4]

- Increased fidelity

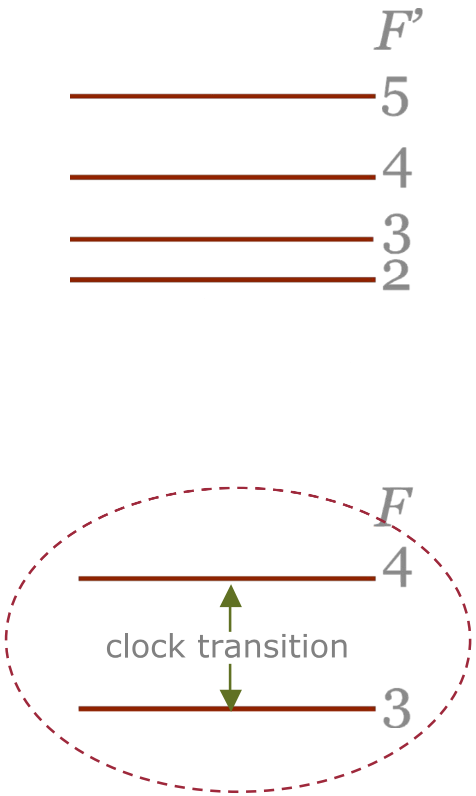
Non classical states for quantum information

- Qubit system

1. *Auzinsh et al*. PRL **93** (17) 173002
2. *Wineland, Bollinger, Itano, and Heinzen*. PRA **50** (1) 67
3. *Andre, Sørensen, and Lukin*. PRL **92**, 230801
4. *Hammerer, Mølmer, Polzik, and Cirac*. PRA **70** 044304

Continuous atomic variables

Pseudo-spin on the Bloch sphere[1,2]



Atomic pseudo-spin

$$\hat{j}_x^{(k)} = \tfrac{1}{2}(\hat{\sigma}_{34}^{(k)} + \hat{\sigma}_{43}^{(k)})$$

$$\hat{j}_y^{(k)} = \tfrac{-i}{2}(\hat{\sigma}_{34}^{(k)} - \hat{\sigma}_{43}^{(k)})$$

$$\hat{j}_z^{(k)} = \tfrac{1}{2}(\hat{\sigma}_{33}^{(k)} - \hat{\sigma}_{44}^{(k)})$$

Collective operators

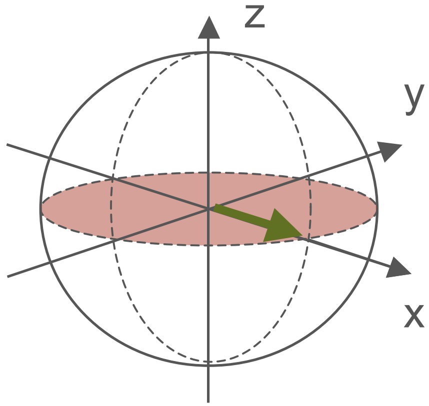
$$\hat{J}_i = \sum_{k \in V} \hat{j}_i^{(k)}$$

$$\hat{N} = \sum_{k \in V} \hat{\sigma}_{33}^{(k)} + \sum_{k \in V} \hat{\sigma}_{44}^{(k)}$$

Coherent spin state

$$\left|\Psi\right\rangle_k = \tfrac{1}{\sqrt{2}}\left|3\right\rangle_k + \tfrac{1}{\sqrt{2}}\left|4\right\rangle_k$$

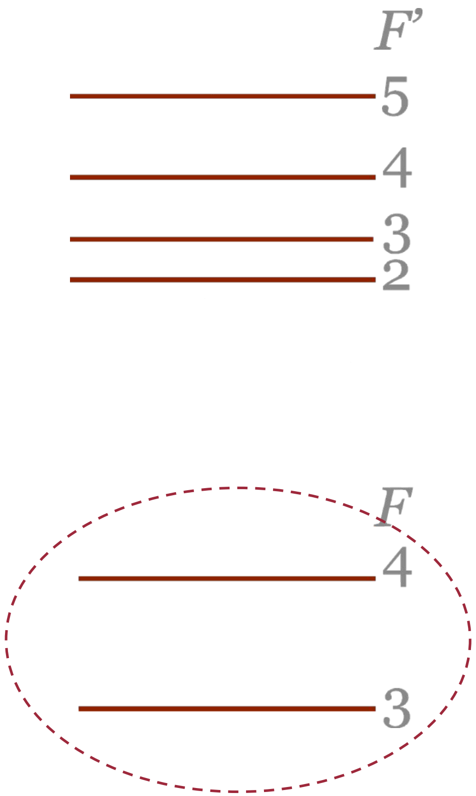
Bloch sphere - qubit



1. *Wineland, Bollinger, Itano, and Heinzen*. PRA **50** (1) p.67
2. *Saffman Oblak, Appel, & Polzik et. al.* preprint available from speaker

Projection noise

Expectation values[1,2]



Mean values

$$\langle \hat{J}_x \rangle = \tfrac{1}{2}\langle \hat{N} \rangle$$

$$\langle \hat{J}_y \rangle = 0$$

$$\langle \hat{J}_z \rangle = 0$$

Uncertainties

$$\langle (\delta \hat{J}_x)^2 \rangle = 0$$

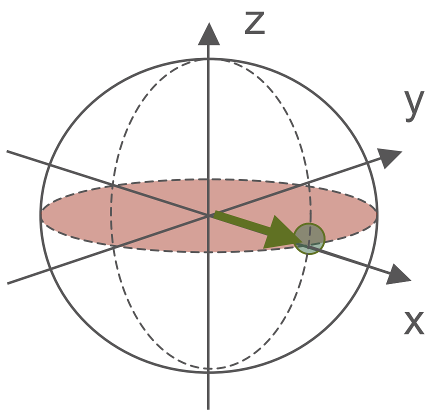
$$\langle (\delta \hat{J}_y)^2 \rangle = \tfrac{1}{2}\langle \hat{J}_x \rangle = \tfrac{1}{4}\langle \hat{N} \rangle$$

$$\langle (\delta \hat{J}_z)^2 \rangle = \tfrac{1}{2}\langle \hat{J}_x \rangle = \tfrac{1}{4}\langle \hat{N} \rangle$$

Coherent spin state

$$|\Psi\rangle_k = \tfrac{1}{\sqrt{2}}|3\rangle_k + \tfrac{1}{\sqrt{2}}|4\rangle_k$$

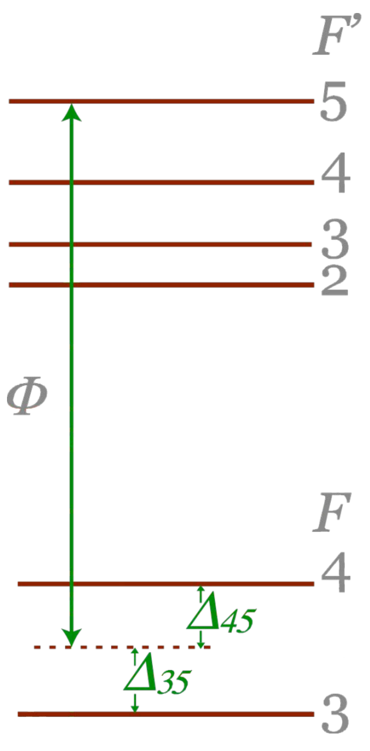
Bloch sphere - qubit

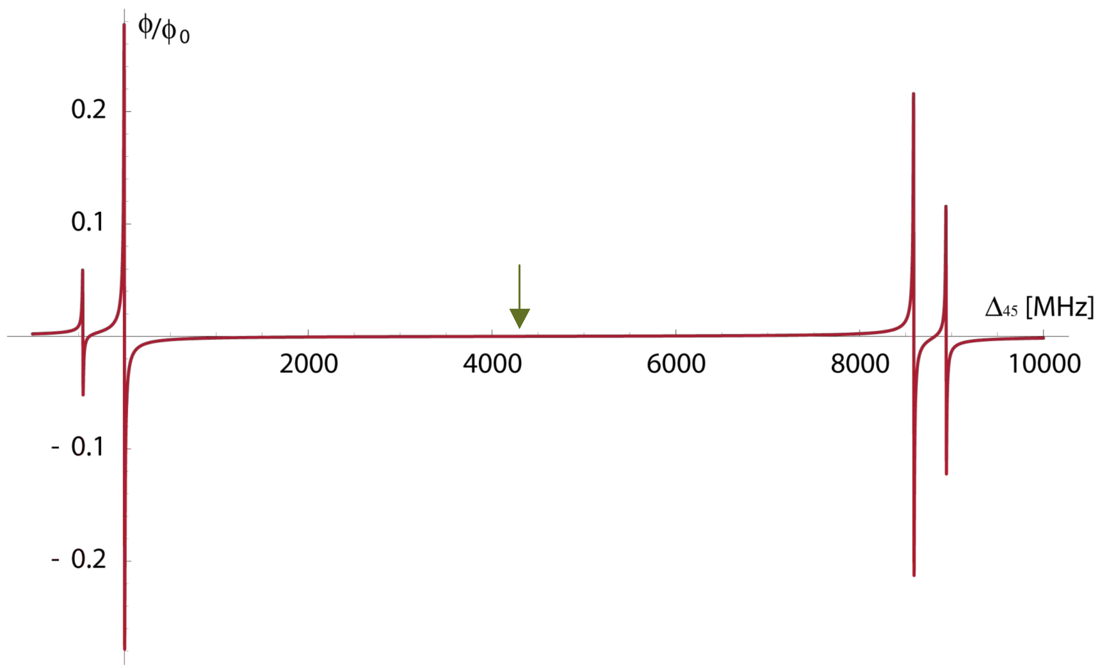


1. *Wineland, Bollinger, Itano, and Heinzen*. PRA **50** (1) p.67
2. *Saffman Oblak, Appel, & Polzik et. al.* preprint available from speaker

Dispersive interaction with light

Phase-shift of probe beams





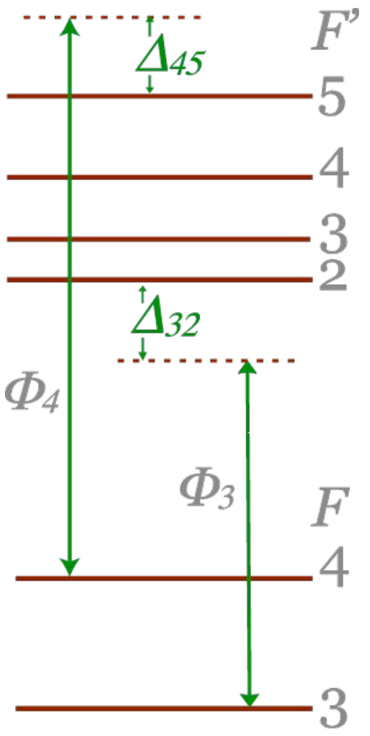
$$\phi = \phi_0 \left[\sum_{e=2,4} c_{3e} \frac{\Delta_{3e}\frac{\gamma}{2}}{\Delta_{3e}^2 + (\frac{\gamma}{2})^2} \hat{N}_3 + \sum_{e=3,5} c_{4e} \frac{\Delta_{4e}\frac{\gamma}{2}}{\Delta_{4e}^2 + (\frac{\gamma}{2})^2} \hat{N}_4 \right]$$

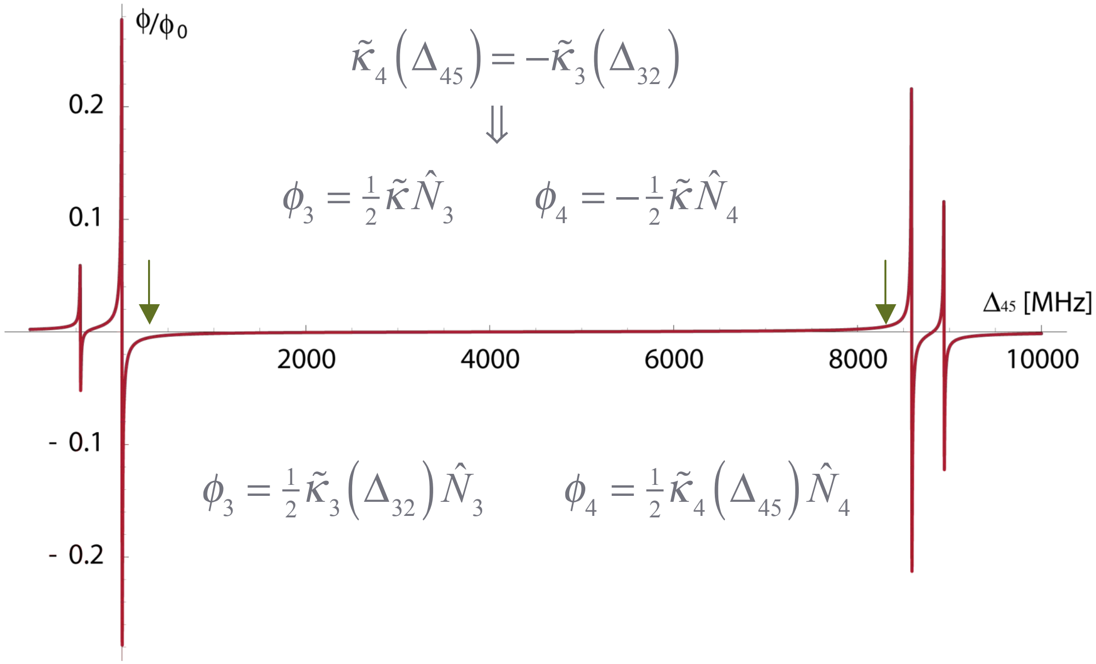
$$= \frac{1}{2}\left[\tilde{\kappa}_3(\Delta_{32})\hat{N}_3 + \tilde{\kappa}_4(\Delta_{45})\hat{N}_4\right]$$

c_{3e} : transition strength, Δ_{3e} : detuning, γ : natural linewidth

Dispersive interaction with light

Phase-shift of probe beams





$$\tilde{\kappa}_4\left(\Delta_{45}\right) = -\tilde{\kappa}_3\left(\Delta_{32}\right)$$

$$\Downarrow$$

$$\phi_3 = \tfrac{1}{2}\tilde{\kappa}\hat{N}_3 \qquad \phi_4 = -\tfrac{1}{2}\tilde{\kappa}\hat{N}_4$$

$$\phi_3 = \tfrac{1}{2}\tilde{\kappa}_3\left(\Delta_{32}\right)\hat{N}_3 \qquad \phi_4 = \tfrac{1}{2}\tilde{\kappa}_4\left(\Delta_{45}\right)\hat{N}_4$$

$$\phi = \phi_0\left[\sum_{e=2,4} c_{3e}\frac{\Delta_{3e}\frac{\gamma}{2}}{\Delta_{3e}^2+\left(\frac{\gamma}{2}\right)^2}\hat{N}_3 + \sum_{e=3,5} c_{4e}\frac{\Delta_{4e}\frac{\gamma}{2}}{\Delta_{4e}^2+\left(\frac{\gamma}{2}\right)^2}\hat{N}_4\right]$$

$$= \tfrac{1}{2}\left[\tilde{\kappa}_3\left(\Delta_{32}\right)\hat{N}_3 + \tilde{\kappa}_4\left(\Delta_{45}\right)\hat{N}_4\right]$$

1. *Petrov, Oblak* et al. PRA 75, 033803 (2007)

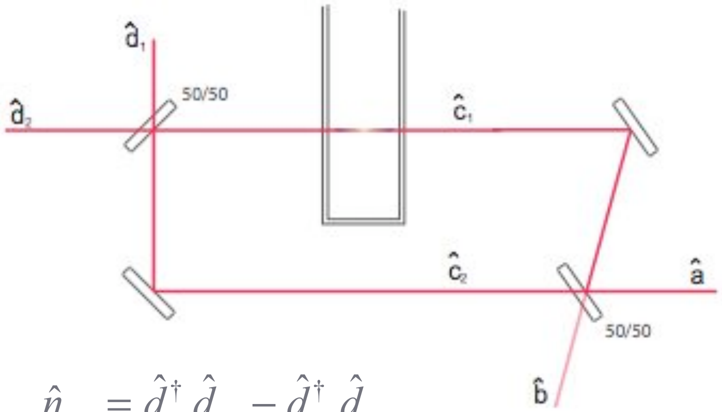
Interferometric measurement

An intuitive picture

$$\tilde{\kappa}_4\left(\Delta_{45}\right) = -\tilde{\kappa}_3\left(\Delta_{32}\right)$$

$$\Downarrow$$

$$\phi_3 = \tfrac{1}{2}\tilde{\kappa}\hat{N}_3 \qquad \phi_4 = -\tfrac{1}{2}\tilde{\kappa}\hat{N}_4$$



$$\hat{n}_{k-} = \hat{d}^{\dagger}_{k1}\hat{d}_{k1} - \hat{d}^{\dagger}_{k2}\hat{d}_{k2}$$

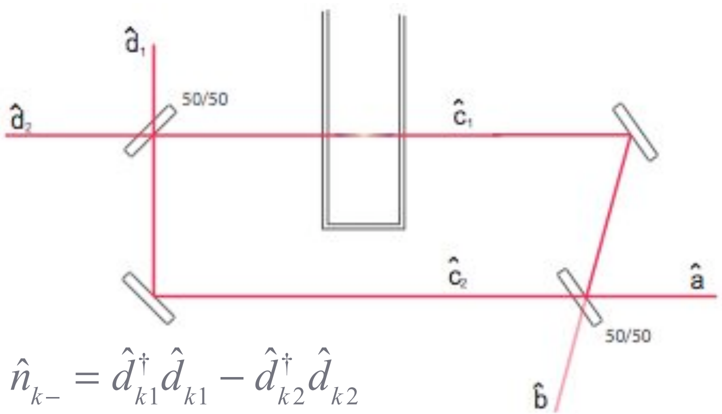
$$i_- \propto \langle \hat{n}_- \rangle \equiv \langle \hat{n}_{3-} + \hat{n}_{4-} \rangle$$
$$= \langle \hat{n}_3 \rangle \cos(\phi_3 + \phi') + \langle \hat{n}_4 \rangle \cos(\phi_4 + \phi')$$
$$\approx 2\tilde{\kappa}_3 \langle \hat{n}_3 \rangle \hat{N}_{3z} - 2\tilde{\kappa}_4 \langle \hat{n}_4 \rangle \hat{N}_{4z}$$
$$= 2\tilde{\kappa}\langle \hat{n} \rangle \left[\hat{N}_{3z} - \hat{N}_{4z}\right] = 2\tilde{\kappa}\tau\langle \hat{n} \rangle \hat{J}_z$$

interferometer optical path-length offset: $\pi/2$

1. *Petrov*, *Oblak* et al. PRA 75, 033803 (2007)

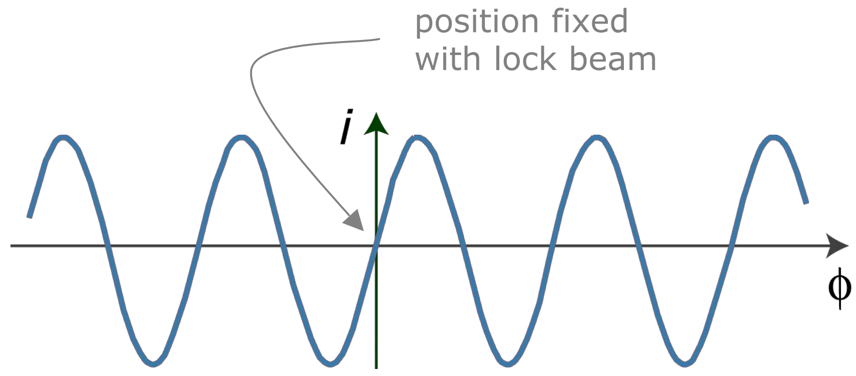
Interferometric measurement

An intuitive picture



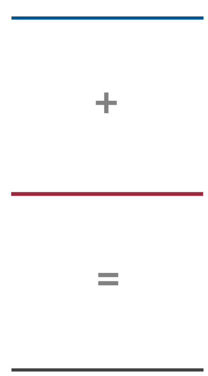
$$\hat{n}_{k-} = \hat{d}^\dagger_{k1}\hat{d}_{k1} - \hat{d}^\dagger_{k2}\hat{d}_{k2}$$

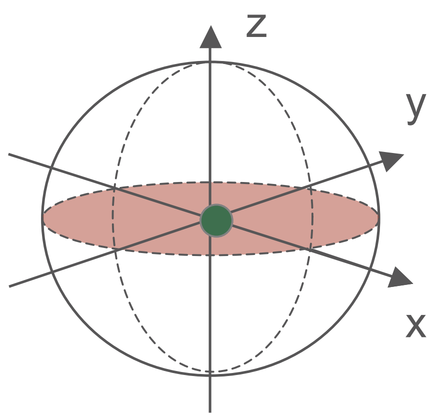
$$
\begin{aligned}
i_- \propto \langle \hat{n}_- \rangle &\equiv \langle \hat{n}_{3-} + \hat{n}_{4-} \rangle \\
&= \langle \hat{n}_3 \rangle \cos(\phi_3 + \phi') + \langle \hat{n}_4 \rangle \cos(\phi_4 + \phi') \\
&\approx 2\tilde{\kappa}_3 \langle \hat{n}_3 \rangle \hat{N}_{3z} + 2\tilde{\kappa}_4 \langle \hat{n}_4 \rangle \hat{N}_{4z} \\
&= 2\tilde{\kappa} \langle \hat{n} \rangle \left[\hat{N}_{3z} - \hat{N}_{4z} \right] = 2\tilde{\kappa}\tau \langle \hat{n} \rangle \hat{J}_z
\end{aligned}
$$



Output signal

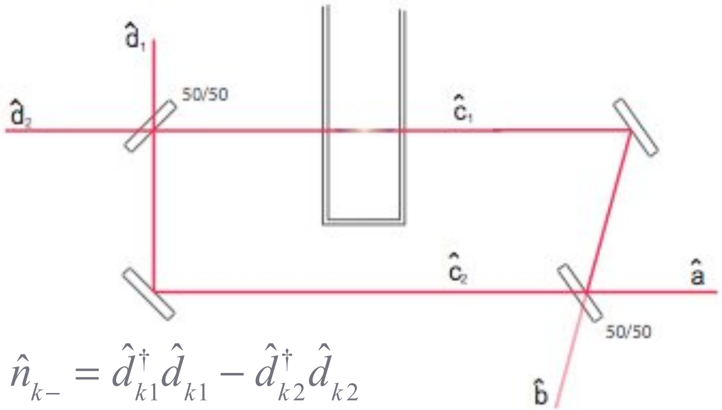
Atomic state



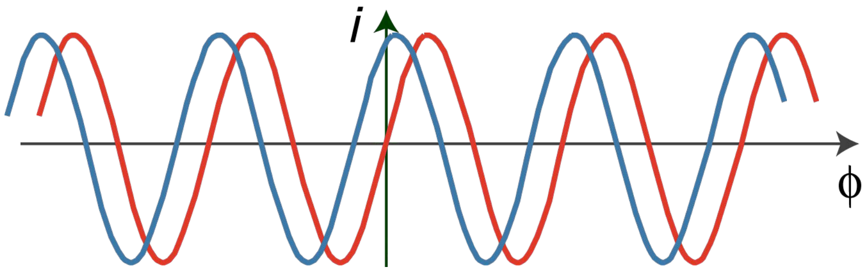


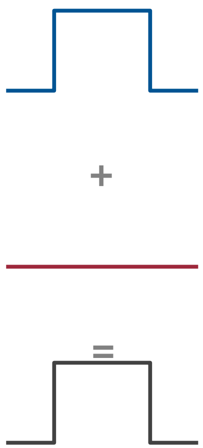
Interferometric measurement

An intuitive picture

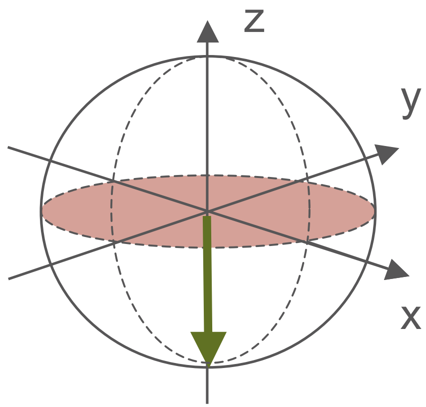


$$\hat{n}_{k-} = \hat{d}^\dagger_{k1}\hat{d}_{k1} - \hat{d}^\dagger_{k2}\hat{d}_{k2}$$



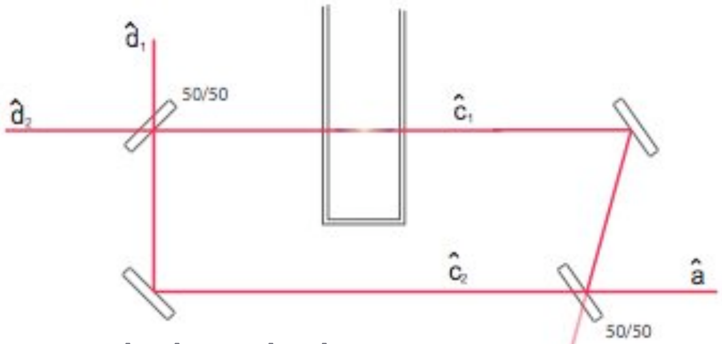


$$\begin{aligned} i_- &\propto \langle \hat{n}_- \rangle \equiv \langle \hat{n}_{3-} + \hat{n}_{4-} \rangle \\ &= \langle \hat{n}_3 \rangle \cos(\phi_3 + \phi') + \langle \hat{n}_4 \rangle \cos(\phi_4 + \phi') \\ &\approx 2\tilde{\kappa}_3 \langle \hat{n}_3 \rangle \hat{N}_{3z} + 2\tilde{\kappa}_4 \langle \hat{n}_4 \rangle \hat{N}_{4z} \\ &= 2\tilde{\kappa} \langle \hat{n} \rangle \left[\hat{N}_{3z} - \hat{N}_{4z} \right] = 2\tilde{\kappa}\tau \langle \hat{n} \rangle \hat{J}_z \end{aligned}$$

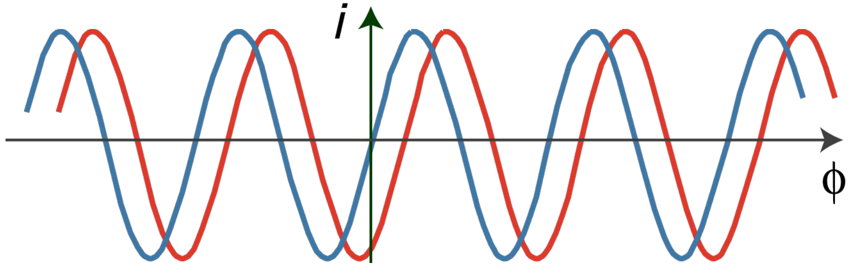


Interferometric measurement

An intuitive picture



$$\hat{n}_{k-} = \hat{d}^{\dagger}_{k1}\hat{d}_{k1} - \hat{d}^{\dagger}_{k2}\hat{d}_{k2}$$

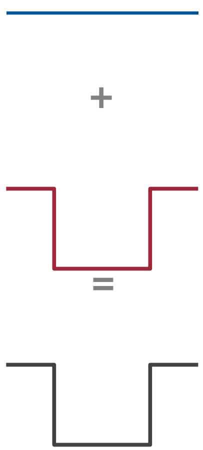


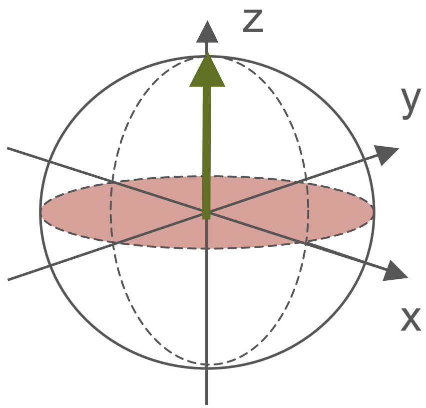
$$i_{-} \propto \langle \hat{n}_{-} \rangle \equiv \langle \hat{n}_{3-} + \hat{n}_{4-} \rangle$$

$$= \langle \hat{n}_3 \rangle \cos(\phi_3 + \phi') + \langle \hat{n}_4 \rangle \cos(\phi_4 + \phi')$$

$$\approx 2\tilde{\kappa}_3 \langle \hat{n}_3 \rangle \hat{N}_{3z} + 2\tilde{\kappa}_4 \langle \hat{n}_4 \rangle \hat{N}_{4z}$$

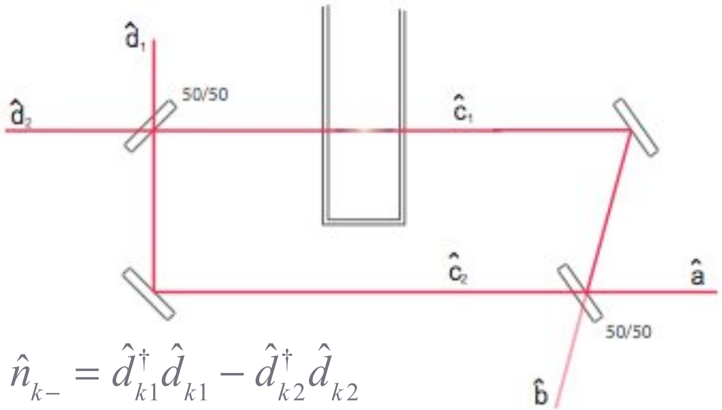
$$= 2\tilde{\kappa} \langle \hat{n} \rangle \left[\hat{N}_{3z} - \hat{N}_{4z} \right] = 2\tilde{\kappa}\tau \langle \hat{n} \rangle \hat{J}_z$$

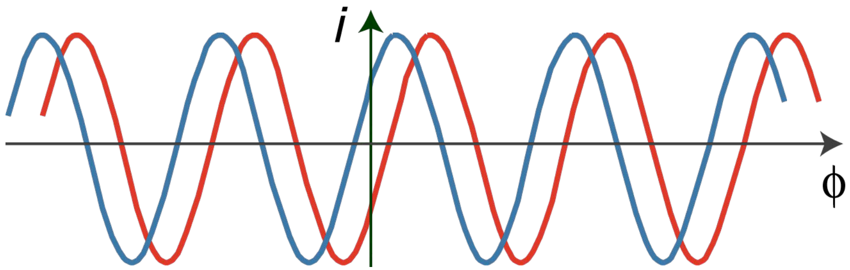




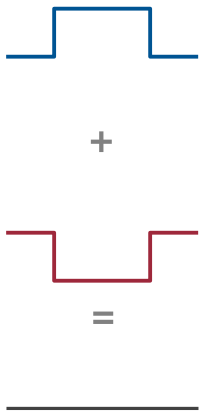
Interferometric measurement

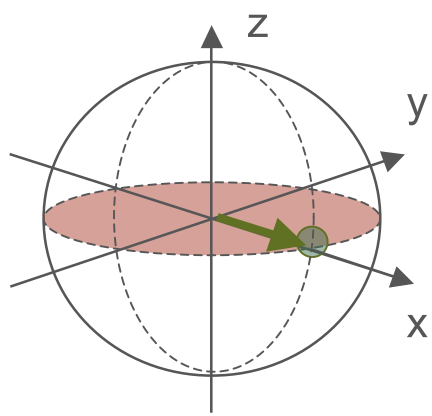
An intuitive picture





$$\hat{n}_{k-} = \hat{d}^{\dagger}_{k1}\hat{d}_{k1} - \hat{d}^{\dagger}_{k2}\hat{d}_{k2}$$

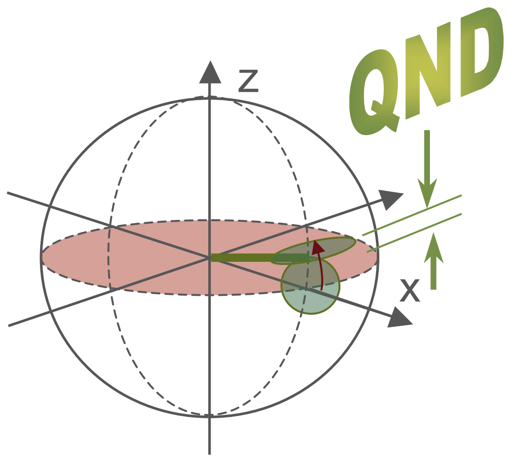




$$\begin{aligned} i_- \propto \langle \hat{n}_- \rangle &\equiv \langle \hat{n}_{3-} + \hat{n}_{4-} \rangle \\ &= \langle \hat{n}_3 \rangle \cos(\phi_3 + \phi') + \langle \hat{n}_4 \rangle \cos(\phi_4 + \phi') \\ &\approx 2\tilde{\kappa}_3 \langle \hat{n}_3 \rangle \hat{N}_{3z} + 2\tilde{\kappa}_4 \langle \hat{n}_4 \rangle \hat{N}_{4z} \\ &= 2\tilde{\kappa} \langle \hat{n} \rangle \left[\hat{N}_{3z} - \hat{N}_{4z} \right] = 2\tilde{\kappa}\tau \langle \hat{n} \rangle \hat{J}_z \end{aligned}$$

Quantum non-demolition measurement

Squeezing by interaction and subsequent measurement of light pulse



Output variances

$$\left(\delta \hat{J}_y^{out}\right)^2 = \left(\delta \hat{J}_y^{in}\right)^2 + \tilde{\kappa}\tau \left\langle \hat{N} \right\rangle \left(\delta \hat{n}_\perp^{in}\right)^2 = \tfrac{1}{4}\left\langle \hat{N} \right\rangle \left(1 + 4\tilde{\kappa}\tau \left\langle \hat{n} \right\rangle\right)$$

$$\left(\delta \hat{J}_y^{out}\right)^2 \left(\delta \hat{J}_z^{out}\right)^2 = \tfrac{1}{16}\left\langle \hat{N} \right\rangle^2 \quad \Rightarrow \quad \left(\delta \hat{J}_z^{out}\right)^2 = \frac{\tfrac{1}{4}\left\langle \hat{N} \right\rangle}{\left(1 + 4\tilde{\kappa}\tau \left\langle \hat{n} \right\rangle\right)}$$

QND interaction Hamiltonian

$$\hat{H}_{\text{int}} = \hbar 4\tilde{\kappa} \tfrac{1}{2}\left(\hat{n} + \hat{n}_\perp\right)\hat{J}_z \quad \Rightarrow$$

$$\hat{n}_{k\perp} = -i(\hat{d}_{k1}^\dagger \hat{d}_{k2} - \hat{d}_{k2}^\dagger \hat{d}_{k1}), \quad \hat{n}_\perp = \hat{n}_{3\perp} + \hat{n}_{4\perp}$$

$$\hat{n}_k = \hat{d}_{k1}^\dagger \hat{d}_{k1} + \hat{d}_{k2}^\dagger \hat{d}_{k2}, \quad \hat{n} = \hat{n}_3 + \hat{n}_4$$

Operator input-output relations

$$n_\perp^{out} = n_\perp^{in} \quad , \quad n_-^{out} = n_-^{in} + 2\tilde{\kappa}\tau \left\langle \hat{n} \right\rangle \hat{J}_z$$

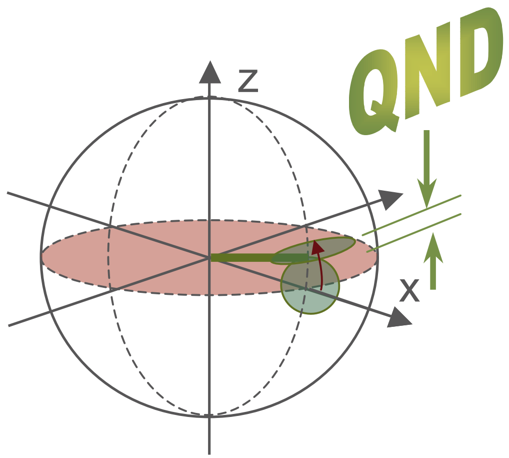
$$\hat{J}_z^{out} = \hat{J}_z^{in} \quad , \quad \hat{J}_y^{out} = \hat{J}_y^{in} + 2\tilde{\kappa}\tau \left\langle \hat{N} \right\rangle \tfrac{1}{2}\left(\left\langle \hat{n} \right\rangle + n_\perp^{in}\right)$$

light-shift

$\left\langle \hat{n}_\perp \right\rangle = 0$, $(\delta \hat{n}_\perp)^2 = \left\langle \hat{n} \right\rangle$, back-action

Quantum non-demolition measurement

Squeezing by interaction and subsequent measurement of light pulse[1]



Output variances[2]

$$\left(\delta \hat{J}_y^{out}\right)^2 = \left(\delta \hat{J}_y^{in}\right)^2 + \tilde{\kappa}\tau\left\langle \hat{N}\right\rangle\left(\delta \hat{n}_\perp^{in}\right)^2 = \tfrac{1}{4}\left\langle \hat{N}\right\rangle\left(1+4\tilde{\kappa}\tau\left\langle \hat{n}\right\rangle\right)$$

$$\left(\delta \hat{J}_y^{out}\right)^2\left(\delta \hat{J}_z^{out}\right)^2 = \tfrac{1}{16}\left\langle \hat{N}\right\rangle^2 \quad \Rightarrow \quad \left(\delta \hat{J}_z^{out}\right)^2 = \tfrac{1}{4}\left\langle \hat{N}\right\rangle \Big/ \left(1+4\tilde{\kappa}\tau\left\langle \hat{n}\right\rangle\right)$$

Squeezing criterion[3,4]

$$\left(\delta \hat{J}_z^{out}\right)^2 < \tfrac{1}{2}\left\langle \hat{J}_x^{out}\right\rangle = \tfrac{1}{4}\eta\left\langle \hat{N}\right\rangle$$

1. *Kuzmich, Bigelow, and Mandel.* Eur. Phys. Lett. **42** 481
2. *Oblak et. al.* PRA **71** 043807
3. *Wineland, Bollinger, Itano, and Heinzen*. PRA **50** (1) 67
4. *Kitagawa and Ueda.* PRA **47** 5138

Setup

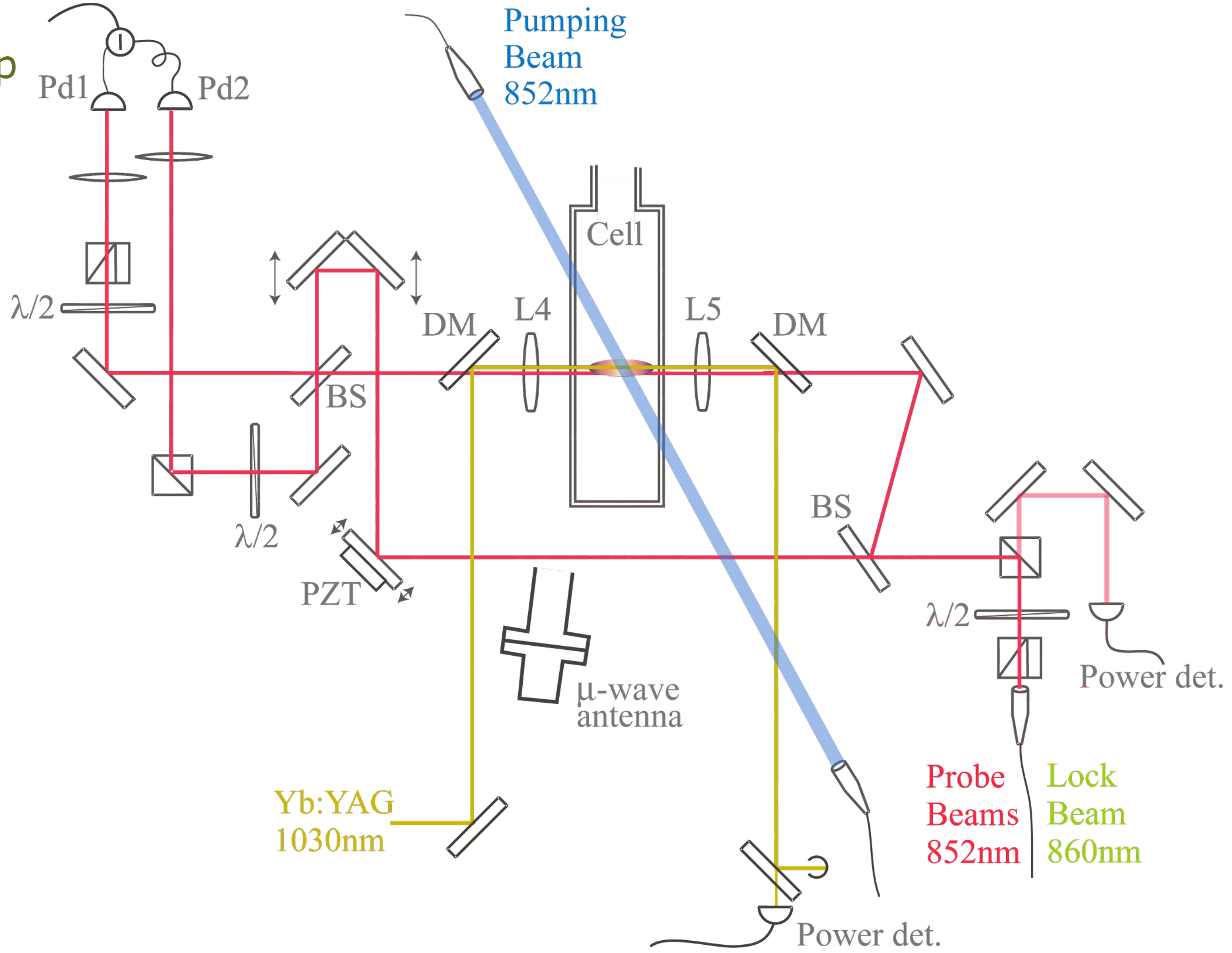
Sketch of setup

White light interferometer

Dipole trap

Optical pumping

Microwaves



Setup

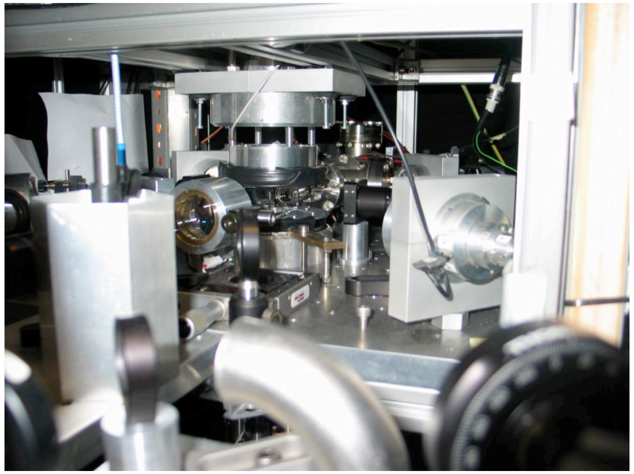
Photos of setup

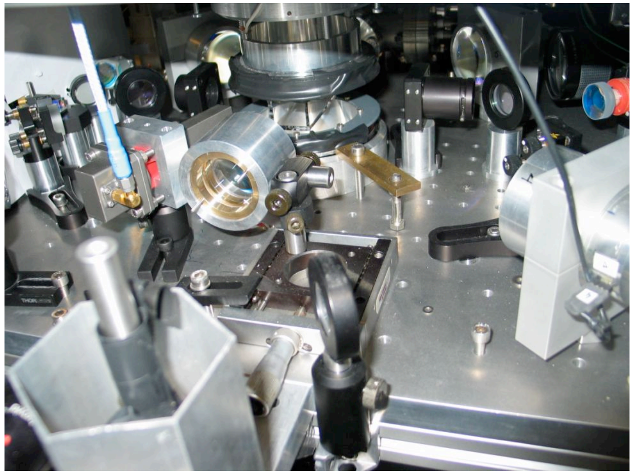
White light interferometer

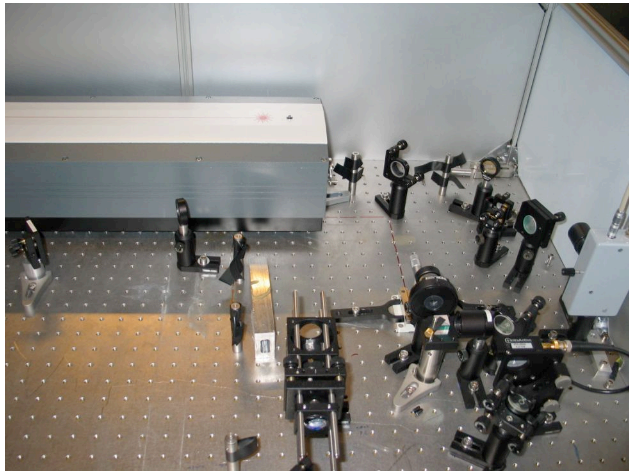
Dipole trap

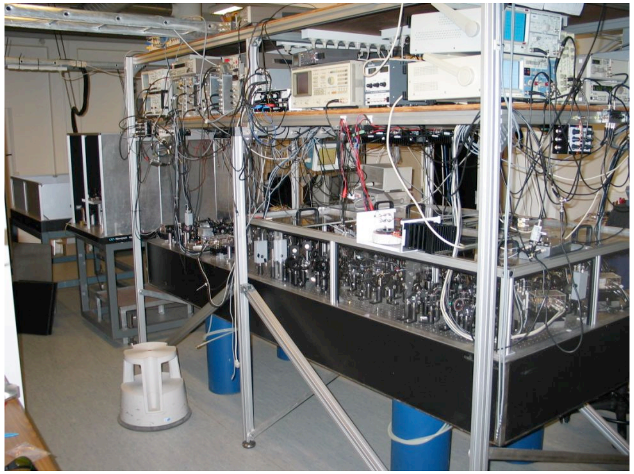
Optical pumping

Microwaves









Measurement procedure

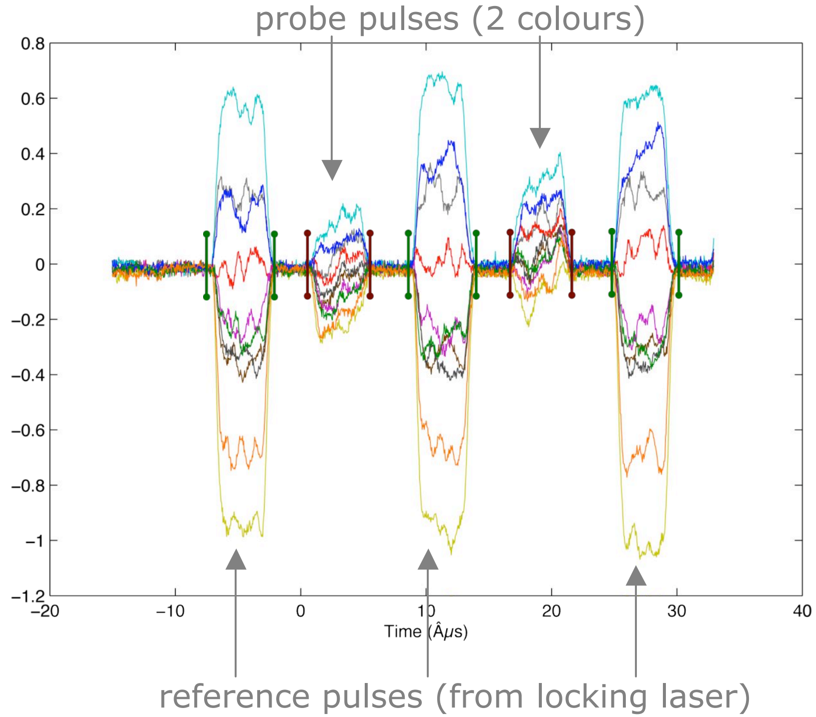
Pulsed measurement[1]

4μs pulses generated in AOM

Incident on balanced low-noise detector

Detector output recorded on digital scope

Integration over pulses (window) and
further processing on PC



1. *Windpassinger, Oblak et al*. New J. Phys. **10** 053032

Measurement procedure

Pulsed measurement - corrections

Baseline subtraction & probe power normalisation

$$p_{j,p}^{(i)} = \rho^{(i)}\left(p_{j,p}^{(i)} - p_{j,bg}^{(i)}\right)$$

$$\rho^{(i)} = \sum_{j}^{M} P_{j,p}^{(i)} \Big/ \frac{1}{KM}\sum_{i,j}^{K,M} P_{j,p}^{(i)}$$

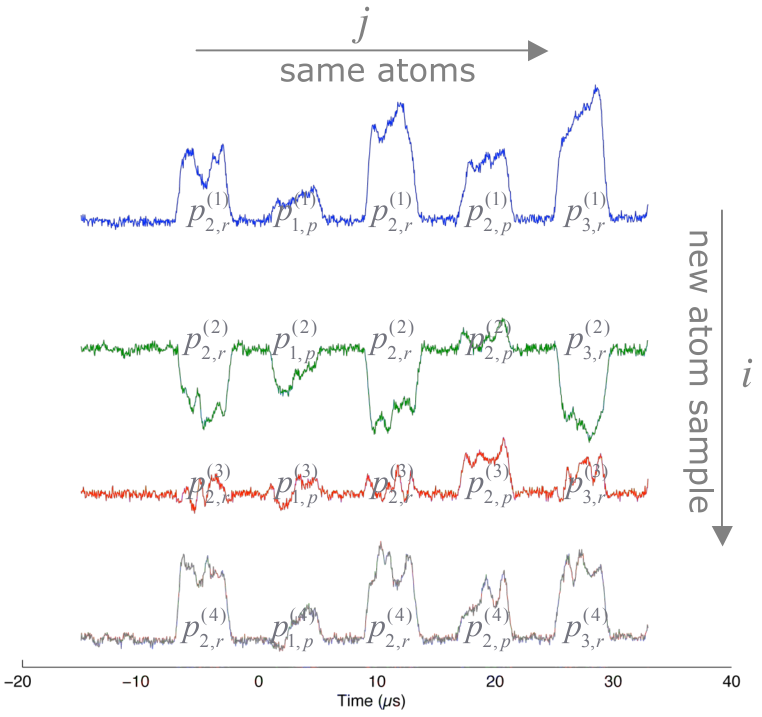
← factor for power normalisation

Reference pulse correction

$$p_{p,j}^{(i)} = p_{p,j}^{(i)} - \chi_{ref}\,\tfrac{1}{2}\left(p_{r,j}^{(i)} + p_{r,j+1}^{(i)}\right),$$

$$\chi_{ref} = \min\left\{\frac{1}{K}\sum_{i=1}^{K}\left[p_{p,j}^{(i)} - \chi\,\tfrac{1}{2}\left(p_{r,j}^{(i)} + p_{r,j+1}^{(i)}\right)\right]^2\right\}$$

correlation factor between probe and reference pulses
- classical fluctuations $\Rightarrow \chi_{ref} \approx 1$
- shot noise $\Rightarrow \chi_{ref} \approx 0$



1. *Windpassinger, Oblak et al.* New J. Phys. **10** 053032

Measurement procedure

Pulsed measurement - noise calculation

Projection noise measurement

$$\left(\delta p_1\right)^2_{proj} = \frac{1}{k-1}\sum_{i=1}^{K}\left[p_{p,1}^{(i)}\right]^2 \propto \frac{1}{4}\left\langle \hat{N}\right\rangle$$

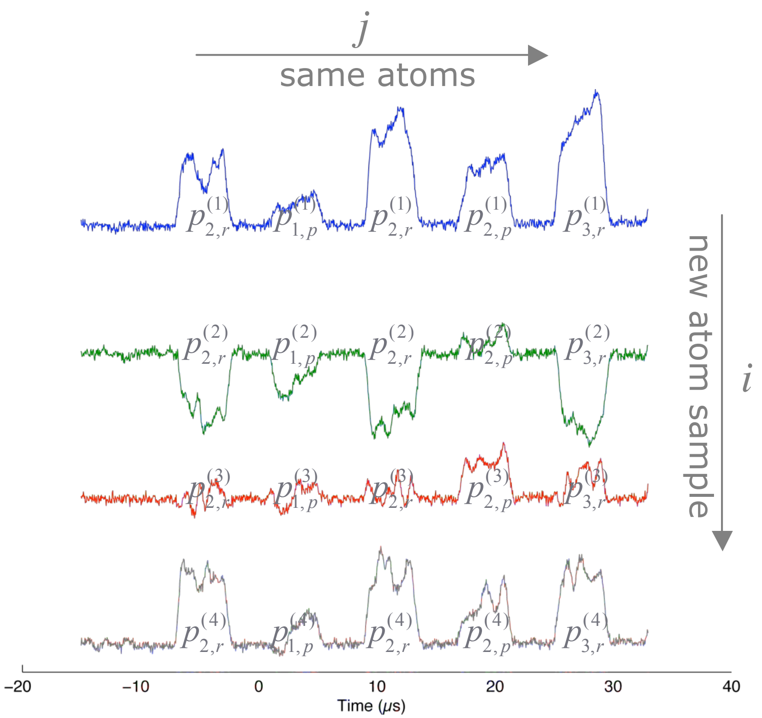
Squeezing measurement - in principle

$$\left(\delta p_{12}\right)^2_{sq} = \frac{1}{2k-1}\sum_{i=1}^{K}\left[p_{p,2}^{(i)} - \chi_{sq} p_{p,1}^{(i)}\right]^2$$

$$\chi_{sq} = \min_{\chi}\left\{\frac{1}{2k-1}\sum_{i=1}^{K}\left[p_{p,2}^{(i)} - \chi p_{p,1}^{(i)}\right]^2\right\}$$

Experimental squeezing criterion

$$\left(\delta p_{12}\right)^2_{sq} < \frac{1}{4}\eta\left\langle \hat{N}\right\rangle = \eta\left(\delta p_1\right)^2_{proj}$$



Pulse durations: 4μs
Pulse separation: 16μs
Probe power: 10 μW ~ 1.6 10^8 photons
Reference power: 50 μW ~ 8 10^8 photons

Near spin-squeezing

Experimental parameters

Projection noise measurement

$$\left(\delta p_1\right)^2_{proj} = \frac{1}{k-1}\sum_{i=1}^{K}\left[p^{(i)}_{p,1}\right]^2 \propto \tfrac{1}{4}\left\langle \hat{N}\right\rangle$$

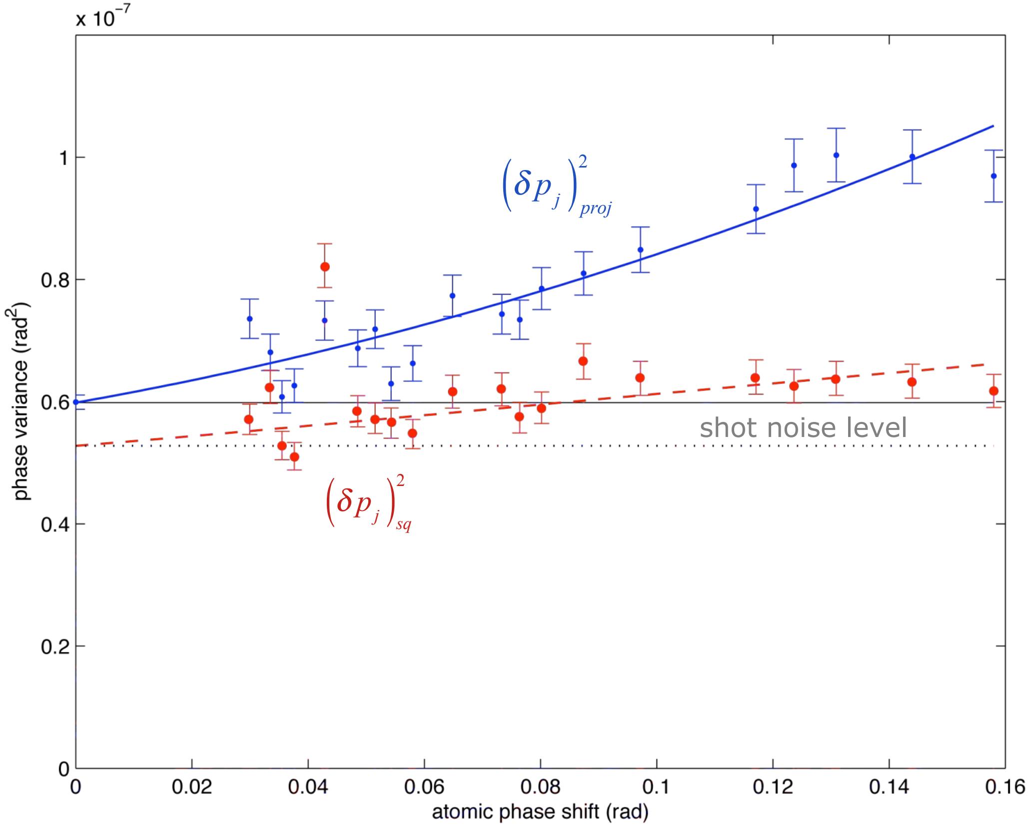
Squeezing measurement - in principle

$$\left(\delta p_j\right)^2_{sq} = \frac{1}{2k-1}\sum_{i=1}^{K}\left[p^{(i)}_{p,j} - \chi_{sq} p^{(i)}_{p,j+1}\right]^2$$

$$\chi_{sq} = \min_{\chi}\left\{\frac{1}{2k-1}\sum_{i=1}^{K}\left[p^{(i)}_{p,j} - \chi p^{(i)}_{p,j+1}\right]^2\right\}$$

Experimental squeezing criterion

$$\left(\delta p_{12}\right)^2_{sq} < \tfrac{1}{4}\eta\left\langle \hat{N}\right\rangle = \eta\left(\delta p_1\right)^2_{proj}$$



Pulse durations: 4μs
Pulse separation: 16μs
Probe power: 10 μW ~ 1.6 10^8 photons
Reference power: 50 μW ~ 8 10^8 photons

Near spin-squeezing

Experimental parameters

Projection noise measurement

$$\left(\delta p_1\right)^2_{proj} = \frac{1}{k-1}\sum_{i=1}^{K}\left[p^{(i)}_{p,1}\right]^2 \propto \tfrac{1}{4}\left\langle \hat{N}\right\rangle$$

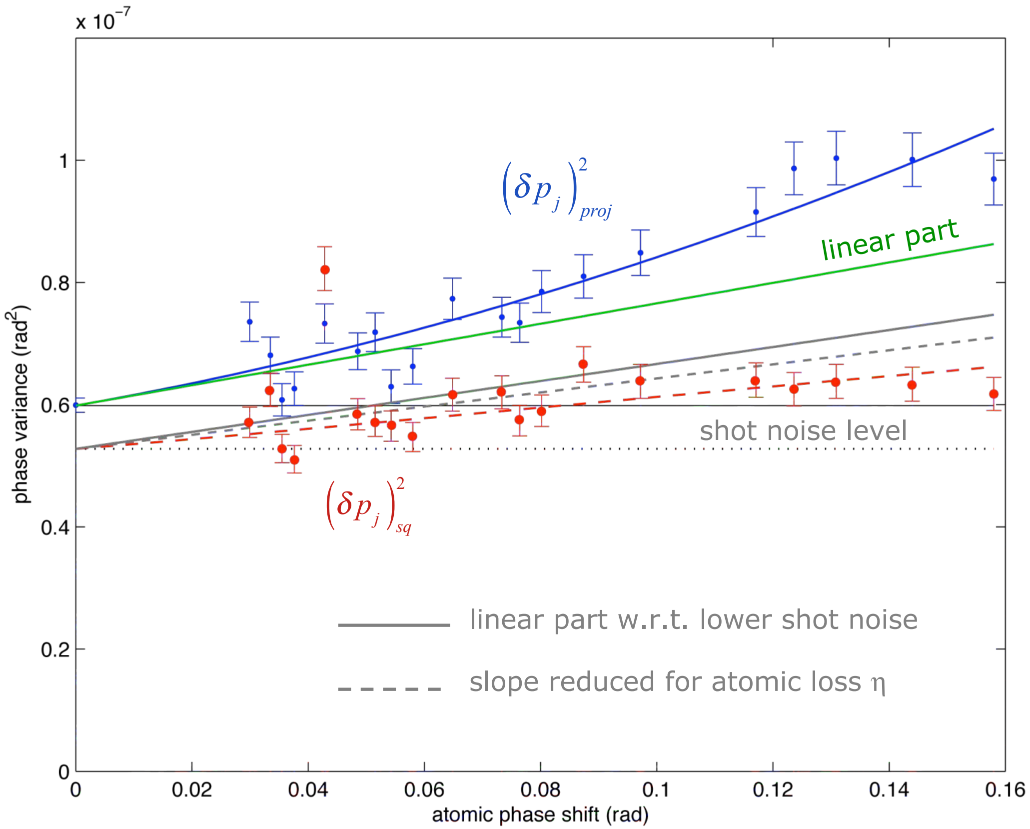
Squeezing measurement - in principle

$$\left(\delta p_j\right)^2_{sq} = \frac{1}{2k-1}\sum_{i=1}^{K}\left[p^{(i)}_{p,j} - \chi_{sq} p^{(i)}_{p,j+1}\right]^2$$

$$\chi_{sq} = \min_{\chi}\left\{\frac{1}{2k-1}\sum_{i=1}^{K}\left[p^{(i)}_{p,j} - \chi p^{(i)}_{p,j+1}\right]^2\right\}$$

Experimental squeezing criterion

$$\left(\delta p_{12}\right)^2_{sq} < \tfrac{1}{4}\eta\left\langle \hat{N}\right\rangle = \eta\left(\delta p_1\right)^2_{proj}$$



Pulse durations: 4μs
Pulse separation: 16μs
Probe power: 10 μW ~ 1.6 10⁸ photons
Reference power: 50 μW ~ 8 10⁸ photons

Near spin-squeezing

Experimental parameters

Projection noise measurement

$$\left(\delta p_1\right)^2_{proj} = \frac{1}{k-1}\sum_{i=1}^{K}\left[p^{(i)}_{p,1}\right]^2 \propto \frac{1}{4}\left\langle \hat{N} \right\rangle$$

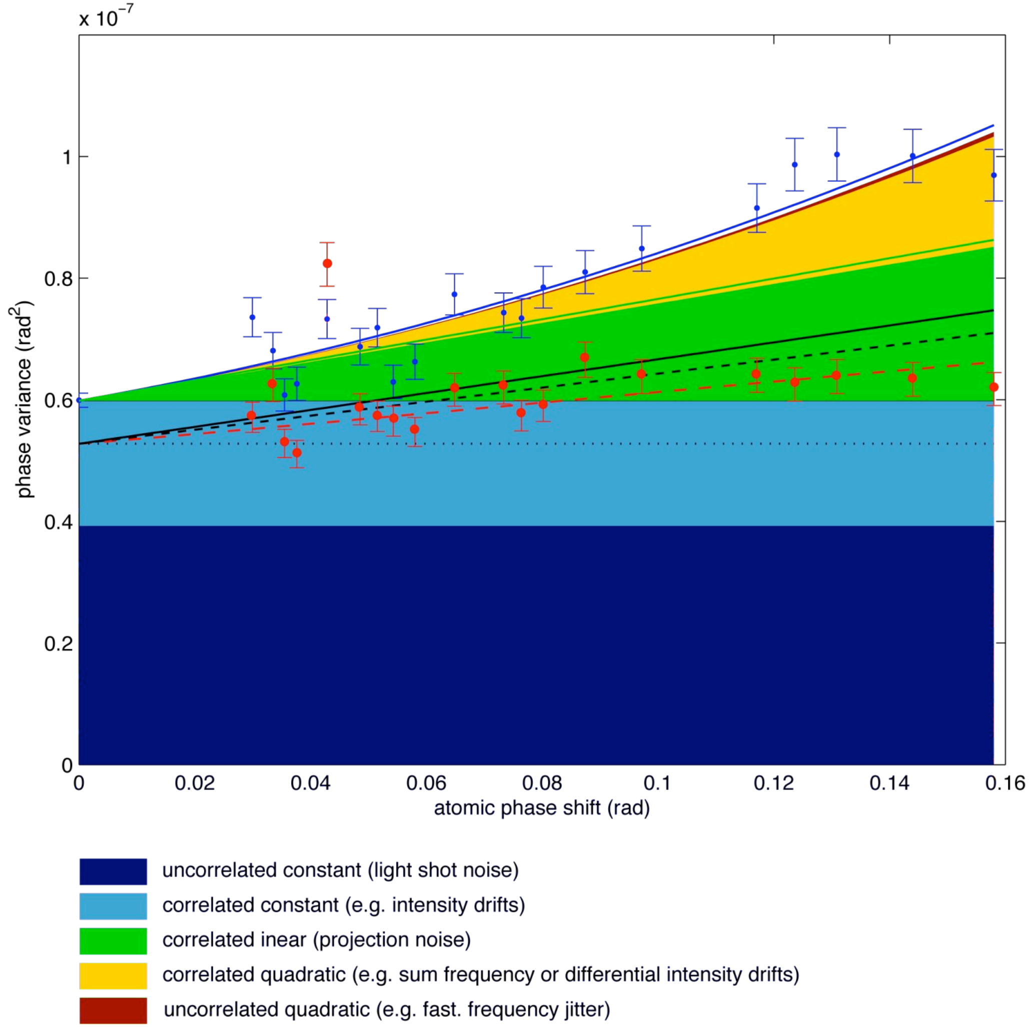
Squeezing measurement - in principle

$$\left(\delta p_j\right)^2_{sq} = \frac{1}{2k-1}\sum_{i=1}^{K}\left[p^{(i)}_{p,j} - \chi_{sq} p^{(i)}_{p,j+1}\right]^2$$

$$\chi_{sq} = \min_{\chi}\left\{\frac{1}{2k-1}\sum_{i=1}^{K}\left[p^{(i)}_{p,j} - \chi p^{(i)}_{p,j+1}\right]^2\right\}$$

Experimental squeezing criterion

$$\left(\delta p_{12}\right)^2_{sq} < \frac{1}{4}\eta\left\langle \hat{N} \right\rangle = \eta\left(\delta p_1\right)^2_{proj}$$



Conclusion

Shot noise limited phase-sensitive measurement

Non destructive measurements of quantum state dynamics

- Light shift

Observation of the reduction of quantum projection noise

- Limited by classical background noise
- QND measurement

Outlook

Two input port interferometer setup[1]

- Sensitivity to difference frequency of 2 colours -> narrower linewidth
- Reduced sensitivity to acoustic noise -> reference pulses superfluous

No more - that should work

1. *Saffman, Oblak, Appel, and Polzik*. manuscript available from speaker

Acknowledgements

The Danish National Research Foundation

- QUANTOP

European Union

- QAUAC
- QOVAQIAL
- QUICOV
- QWAP

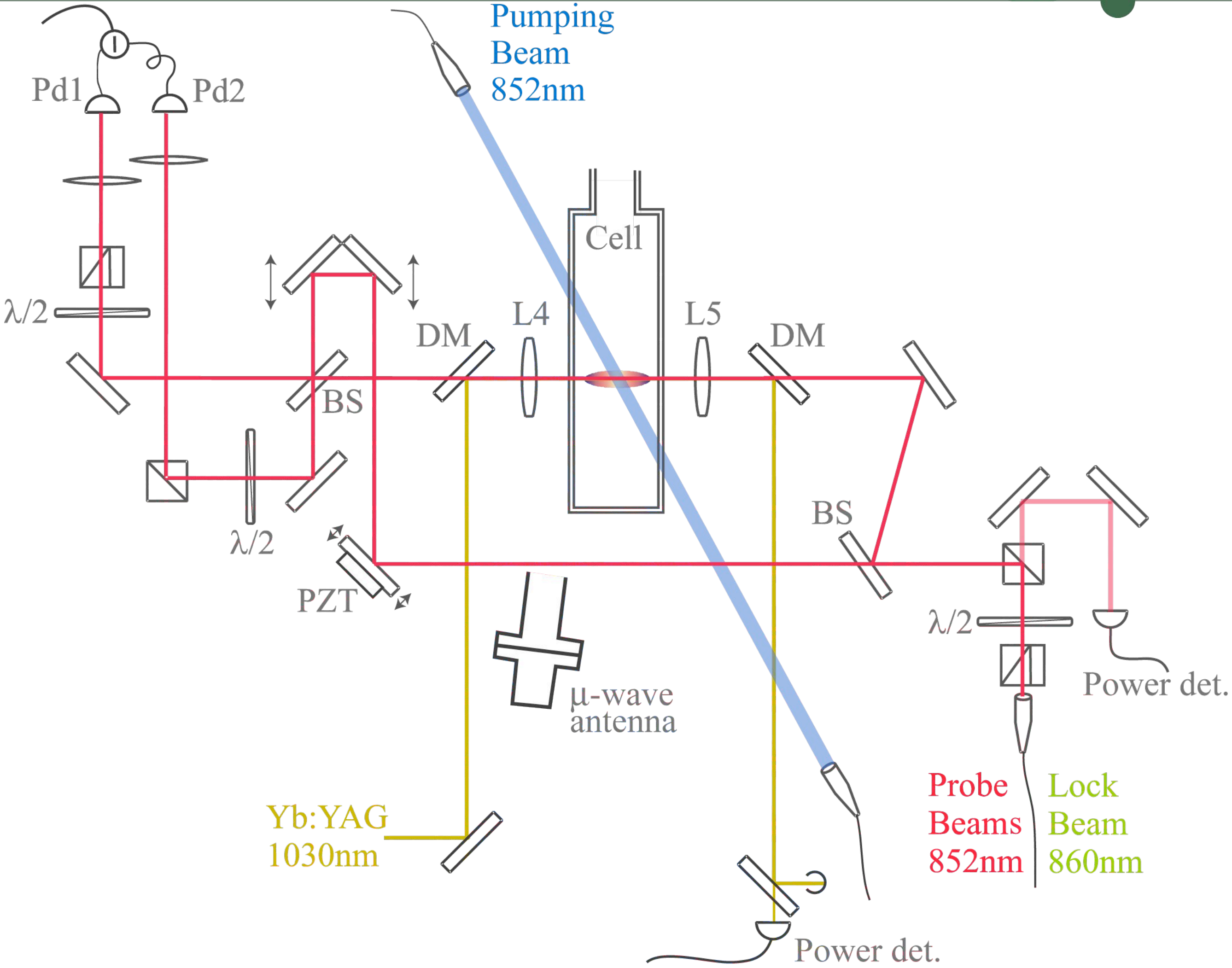
And thanks for your attention

And sorry for speaking too long...

Any questions?

Two input interferometer

Benefits:

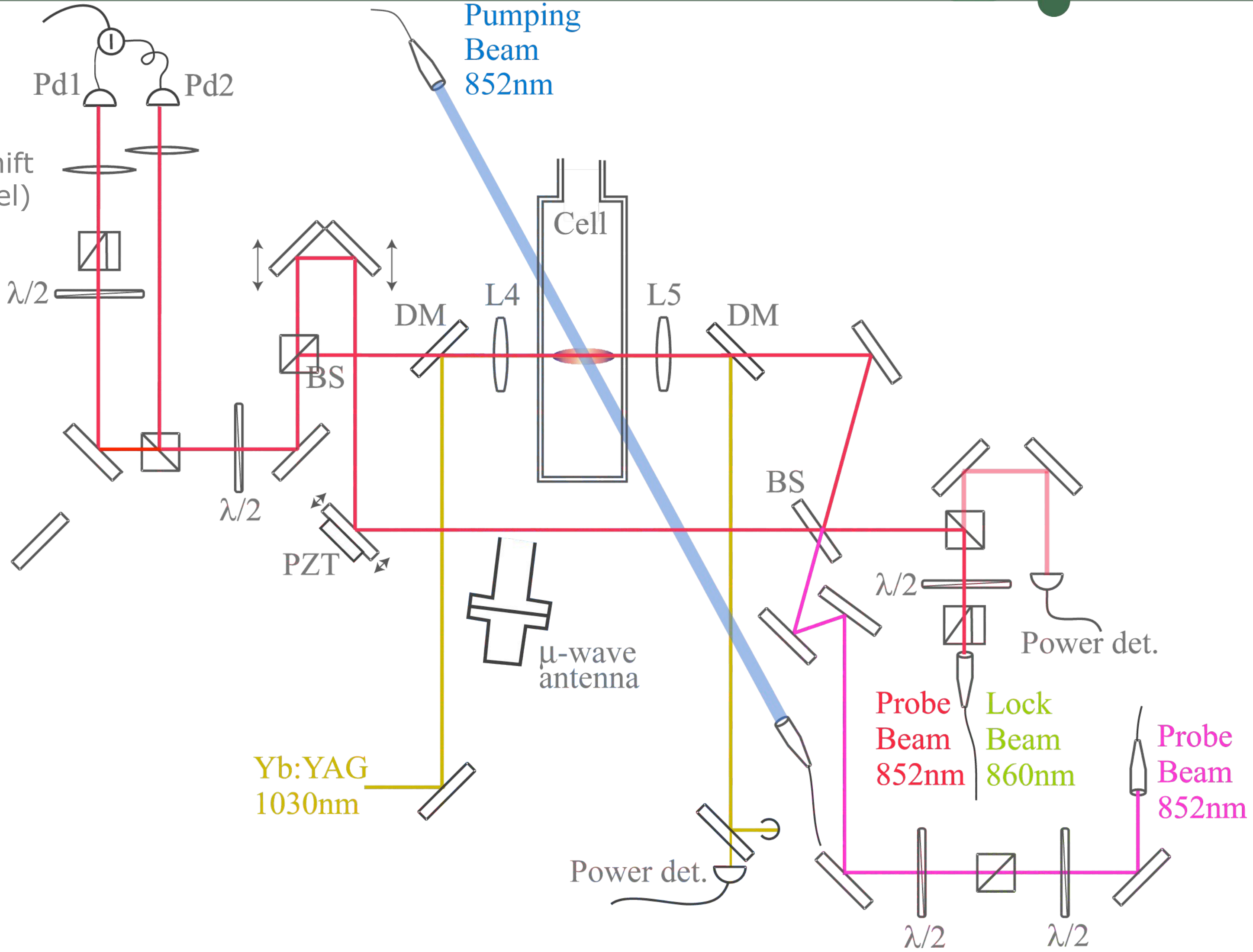


Two input interferometer

Benefits:

Eliminates light shift (two colours cancel)

Insensitive to mechanical and acoustic noise



Noise sources in atomic measurements

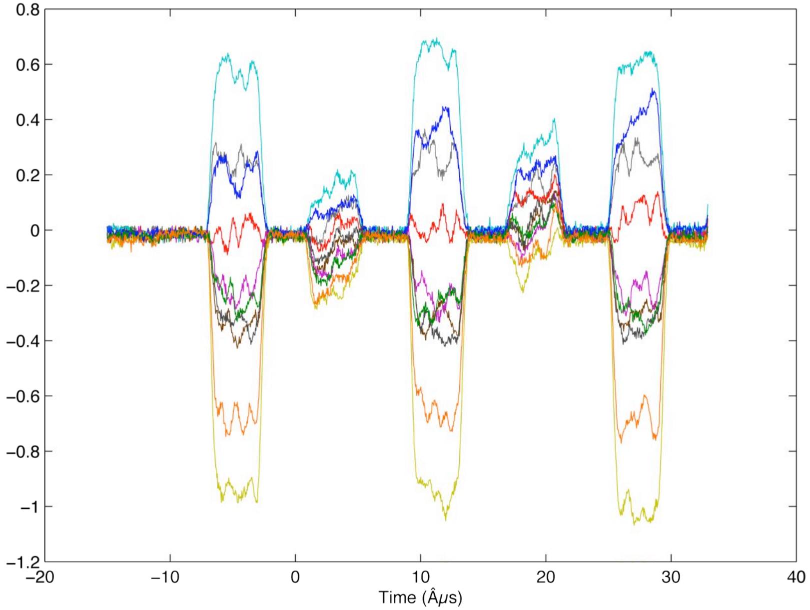
Quantum and classical noise sources

Noise in output signal

$$\hat{n}_-^{out} = \hat{n}_-^{in} + 4\tilde{\kappa}\tau\langle\hat{n}\rangle\hat{J}_z$$

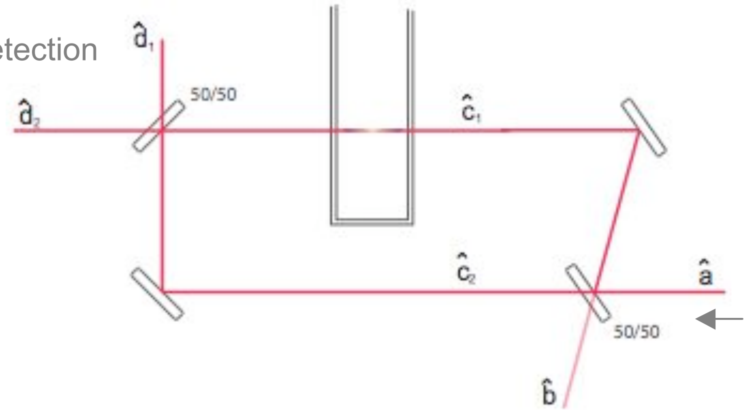
$$(\delta\hat{n}_-^{out})^2 = (\delta\hat{n}_-^{in})^2 + 4\tilde{\kappa}\tau\langle\hat{n}\rangle(\delta\hat{J}_z)^2$$
$$= \langle\hat{n}\rangle + \tilde{\kappa}\tau\langle\hat{n}\rangle\langle\hat{N}\rangle + K\langle\hat{N}\rangle^2 + E$$

light shot | atomic projection | classical garbage | electr.



X Interferometric measurement

Interaction cast in continuous light variables



Schwinger representation

$$\hat{S}_{kx} = \tfrac{1}{2}(\hat{a}_k^\dagger \hat{b}_k + \hat{b}^\dagger \hat{a}_k)$$
$$\hat{S}_{ky} = \tfrac{-i}{2}(\hat{a}_k^\dagger \hat{b}_k - \hat{b}_k^\dagger \hat{a}_k)$$
$$\hat{S}_{kz} = \tfrac{1}{2}(\hat{a}_k^\dagger \hat{a}_k - \hat{b}_k^\dagger \hat{b}_k)$$
$$\hat{n}_k = \hat{a}_k^\dagger \hat{a}_k + \hat{b}_k^\dagger \hat{b}_k$$

$$\langle \hat{a}_k^\dagger \hat{a}_k \rangle = \langle \hat{n}_k \rangle$$
$$\langle \hat{b}_k^\dagger \hat{b}_k \rangle = 0$$
$$\langle \hat{S}_{kx}^c \rangle = \langle \hat{S}_{kz}^c \rangle = 0$$
$$\langle \hat{S}_{ky}^c \rangle = \tfrac{1}{2} \langle \hat{n}_k \rangle$$

Interaction Hamiltonian

$$\hat{H}_{k\,\mathrm{int}} = \hbar 4 \tilde{\kappa}_k \left(\tfrac{1}{2} \hat{n}_k + \hat{S}_{kz}^c \right) \hat{N}_k$$
$$\hat{H}_{\mathrm{int}} = \hbar 4 \Big[\tilde{\kappa}_3 \left(\tfrac{1}{2} \hat{n}_3 + \hat{S}_{3z}^c \right) \hat{N}_3 + \tilde{\kappa}_4 \left(\tfrac{1}{2} \hat{n}_4 + \hat{S}_{4z}^c \right) \hat{N}_4 \Big]$$

$$\Rightarrow$$

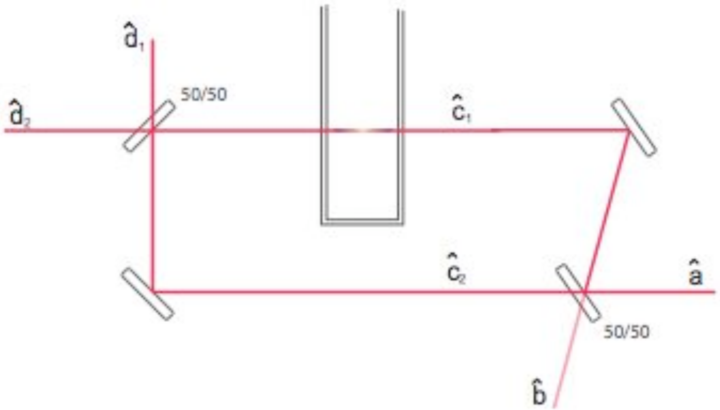
Operator input-output relations

$$\hat{S}_{kz}^{c,out} = \hat{S}_{kz}^{c,in} \quad , \quad \hat{S}_{kx}^{c,out} = \hat{S}_{kx}^{c,in} + 2\tilde{\kappa}_k \tau \langle \hat{n}_k \rangle \hat{N}_k$$
$$\hat{J}_z^{c,out} = \hat{J}_z^{c,in} \quad , \quad \hat{J}_y^{out} = \hat{J}_y^{in} + 2\tilde{\kappa}_k \tau \hat{N}_k \left(\tfrac{1}{2} \langle \hat{n}_k \rangle + \hat{S}_{kz}^c \right)$$

X Interferometric measurement

Quantum picture

$$i_- \propto \left\langle \hat{S}^d_{3z} + \hat{S}^d_{4z} \right\rangle = \hat{S}^{c,out}_{3x} + \hat{S}^{c,out}_{4x} = \hat{S}^{c,in}_{kx} + 2\tilde{\kappa}_k \tau \left\langle \hat{n}_k \right\rangle \hat{J}_z$$

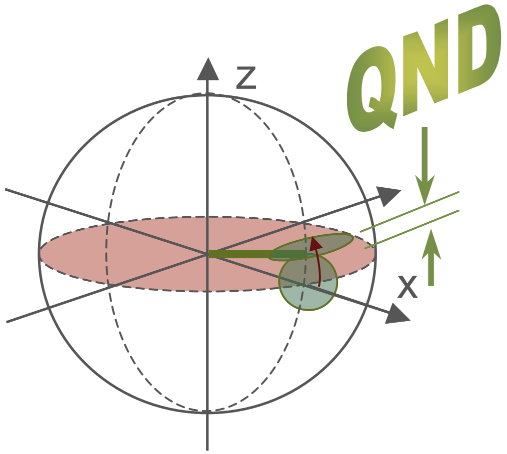


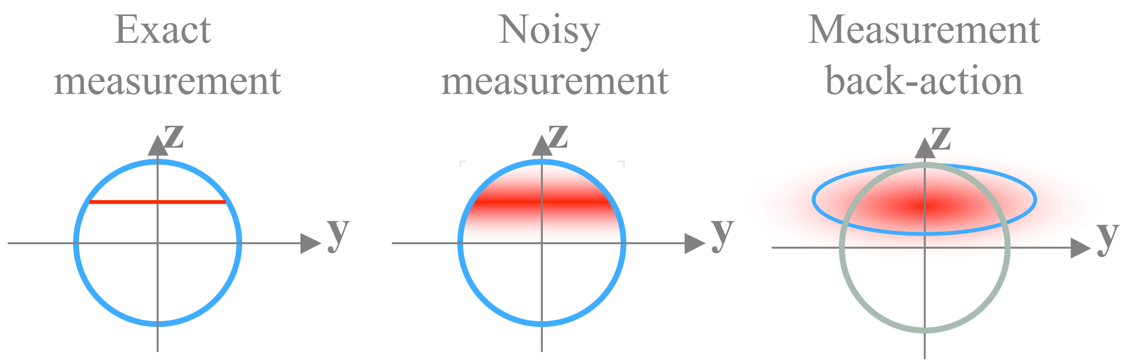
$$\hat{S}^{c,out}_{kz} = \hat{S}^{c,in}_{kz} \quad , \quad \hat{S}^{c,out}_{kx} = \hat{S}^{c,in}_{kx} + 2\tilde{\kappa}_k \tau \left\langle \hat{n}_k \right\rangle \hat{J}_z$$

$$\hat{J}^{c,out}_{z} = \hat{J}^{c,in}_{z} \quad , \quad \hat{J}^{out}_{y} = \hat{J}^{in}_{y} + 2\tilde{\kappa}_k \tau \hat{N} \left(\tfrac{1}{2} \left\langle \hat{n}_k \right\rangle + \hat{S}^{c}_{kz} \right)$$

X Quantum non-demolition measurement

Squeezing by interaction and subsequent measurement of light pulse





QND interaction Hamiltonian

$$\hat{H}_{\mathrm{int}} = \hbar 4\tilde{\kappa}\left(\frac{\hat{n}}{2} + \hat{S}_z^c\right)\hat{J}_z \qquad \Rightarrow$$

Operator input-output relations

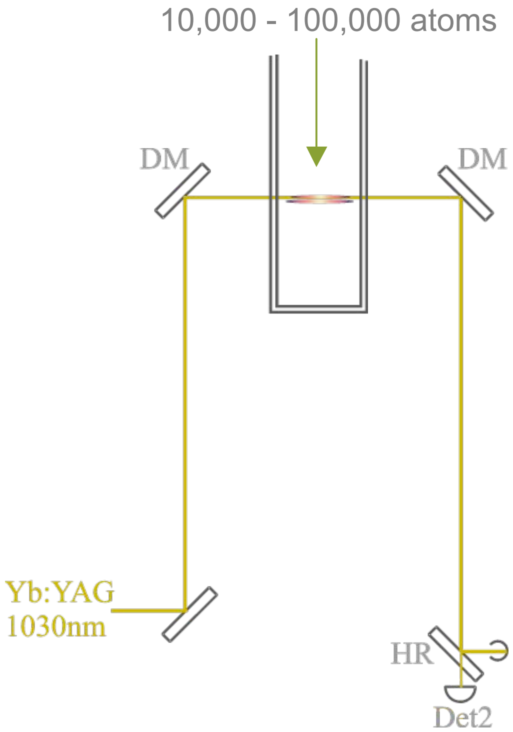
$$\hat{S}_z^{c,out} = \hat{S}_z^{c,in} \quad , \quad \hat{S}_x^{c,out} = \hat{S}_x^{c,in} + 2\tilde{\kappa}\tau\langle\hat{n}\rangle\hat{J}_z$$

$$\hat{J}_z^{c,out} = \hat{J}_z^{c,in} \quad , \quad \hat{J}_y^{out} = \hat{J}_y^{in} + 2\tilde{\kappa}\tau\hat{N}\left(\frac{\langle\hat{n}\rangle}{2} + \hat{S}_z^c\right)$$

Far off-resonant trap (FORT)

Setup

Atoms trapped in MOT
-> sub-doppler cooled
-> loaded into FORT



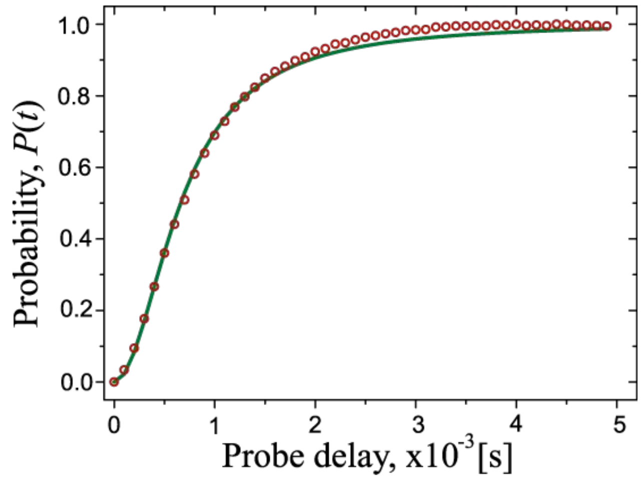
Expansion rate
-> temperature

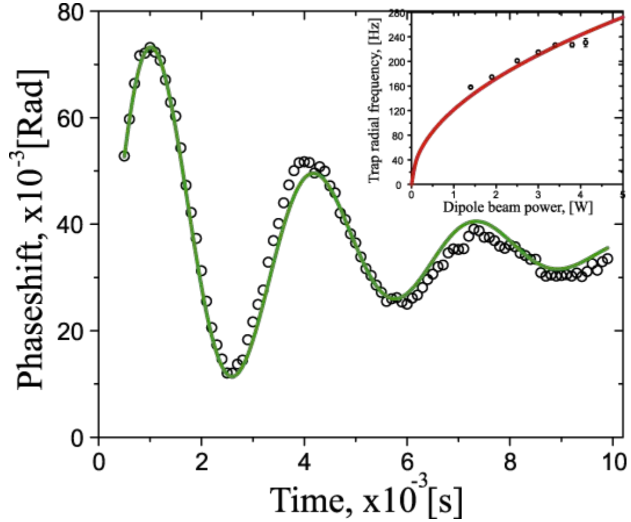
T ≈ 15μK

Only 1 probe colour!

Atom breathing
-> trap radial
oscillation frequency

f = 180 - 220Hz

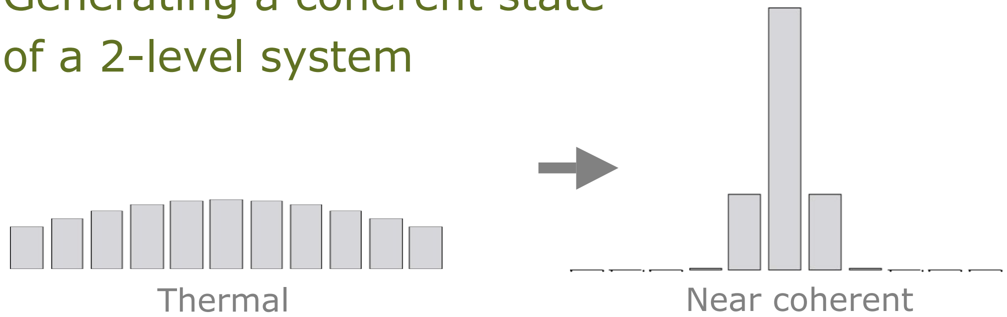


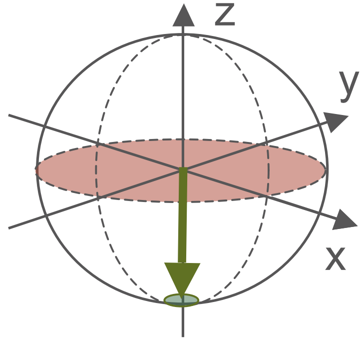


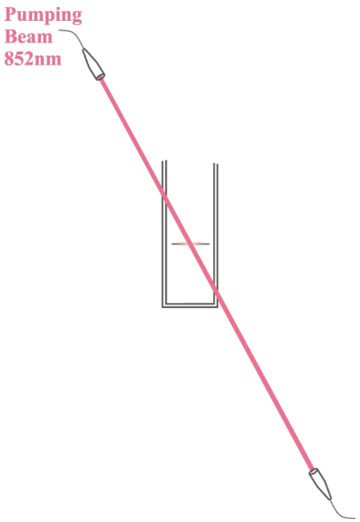
Optical pumping to the $m_f=0$ sublevel

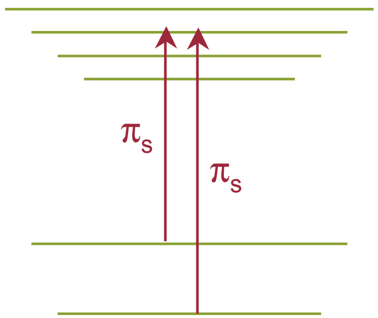
Setup

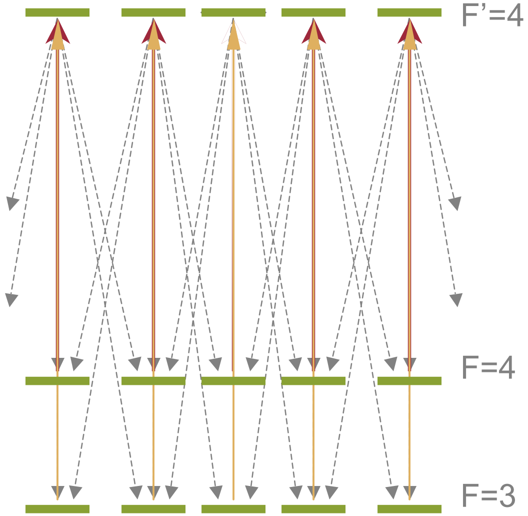
Generating a coherent state of a 2-level system









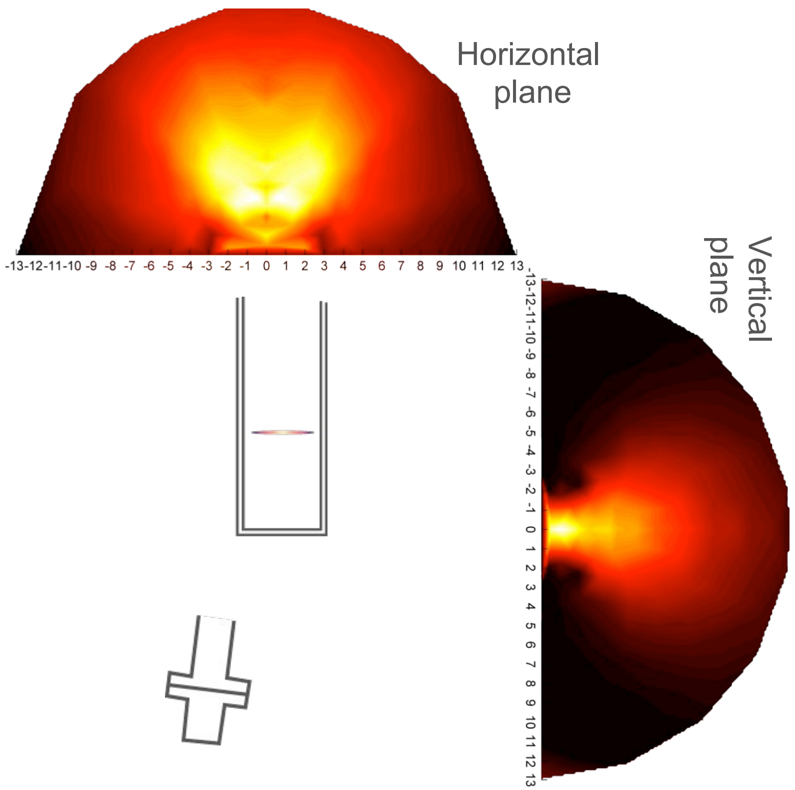


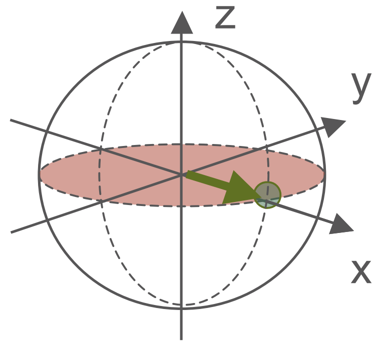
Microwave transitions

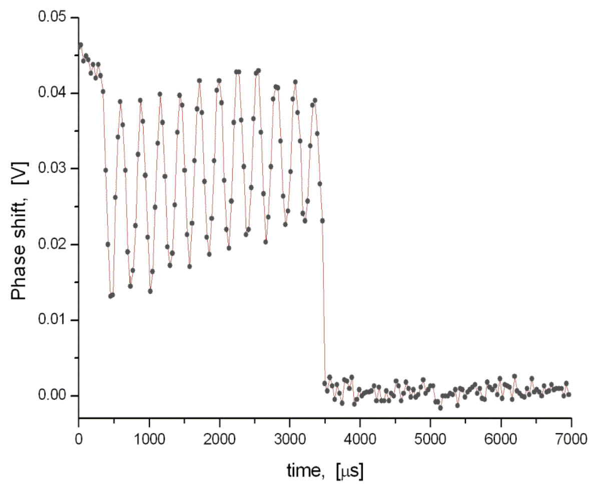
Setup

Magnetic dipole transitions btw hyperfine ground states

Sawed-off waveguide







Optical pumping efficiency:
fraction of atoms in Rabi oscillations,